18MAFA0	ALGORITHMIC GRAPH THEORY

Category	L	T	P	Credit
FE	3	0	0	3

Preamble

An Engineering student needs to have some basic mathematical tools and techniques to apply in diverse applications in Engineering. This emphasizes the development of rigorous logical thinking and analytical skills of the student and appraises him the complete procedure for solving different kinds of problems that occur in engineering. Combinatorial arguments are made a little easier by the use of pictures of the graphs. Natural form of graphs is a set with logical or hierarchical sequencing, such as computer flow charts. The concept of Graph Theory has wide range of applications in Networks, computer architecture, compiling techniques, model checking, artificial intelligence, software engineering, expert systems, software/hardware correctness problem, , DBMS, designing concepts, storage methods etc. This course is designed to provide expertise in algorithmic approach to Graph theory concepts.

Prerequisite

-

Course Outcomes

On the successful completion of the course students will be able to

CO Number	Course Outcome Statement	Weightage*** in %
CO1	Characterize the different graph structures.	25 %
CO2	Construct trees and spanning trees.	20%
CO3	Finding Blocks for connected graph	15%
CO4	Find a maximum matching for a connected graph	15%
CO5	Find a maximum clique for a connected graph and Construct the chromatic Polynomial for the given graph.	15%
CO6	Demonstrate Planar graph and also Euler's formula	10%

CO Mapping with CDIO Curriculum Framework

CO #	TCE Proficiency Scale	Learning Domain Level			CDIO Curricular Components (X.Y.Z)
		Cognitive	Affective	Psychomotor	
CO1	TPS3	Apply	Value	-	1.1.1, 1.2.7, 2.1.1
CO2	TPS3	Apply	Value	-	1.1.1, 1.2.7, 2.1.1
CO3	TPS3	Apply	Value	-	1.1.1 , 1.2.7
CO4	TPS3	Apply	Value	-	1.1.1, 1.2.7
CO5	TPS3	Apply	Value	-	1.1.1, 1.2.7
CO6	TPS2	Understand	Respond	-	1.1.1

Mapping with Programme Outcomes and Programme Specific Outcomes

Cos	PO1	PO2	PO3	PO4	PO5	PO6	PO7	PO8	PO9	PO10	PO11	PO12	PSO1	PSO2	PSO3
CO1	S	S	M	M	-	-	-	-	-	-	-	M			
CO2	S	S	S	S	-	-	-	-	-	-	-	M			
CO3	S	S	M	L	-	-	-	-	-	-	-	M			
CO4	S	S	M	L	-	-	-	-	-	-	-	M			
CO5	S	S	S	M	-	-	-	-	-	-	-	M			
CO6	S	S	S	M	-	-	-	-	-	-	-	M			

S- Strong; M-Medium; L-Low

Assessment Pattern: Cognitive Domain

Cognitive Levels	Continuous Assessment Tests			Assignment			Terminal Examination
	1	**2**	**3**	1	2	3	
Remember	10	10	10	-	-	-	-
Understand	20	20	20	-	-	-	30
Apply	70	70	70	100	100	100	70
Analyse							
Evaluate							
Create							

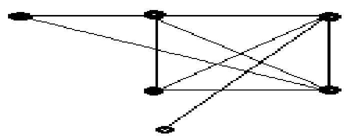
Sample Questions for Course Outcome Assessment**

** (2 to 3 at the cognitive level of course outcome)

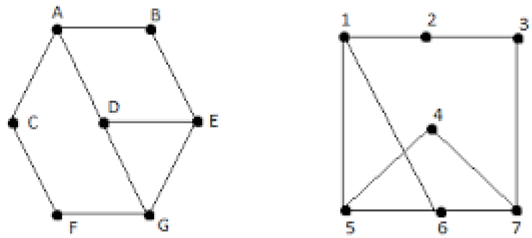
Course Outcome 1(CO1):

1. (i) In each class below, determine the smallest n such that there exist nonisomorphic n-vertex graphs having the same list of vertex degrees.
   (a) All graphs (b) loopless graphs (c) simple graphs

   (ii) Verify whether the following graphs are Eulerian or Hamiltonian graphs



2. Determine whether the following graphs are isomorphic or not.



3. If G has D as a degree sequence, then what do you say about the degree sequence of its complement.

Course Outcome 2(CO2):

1. Show that G has e-|G| +1 fundamental Cycles with respect to any spanning tree
2. Define different types of tree and show that it is a connected graph.
3. Write down BFS algorithm for finding spanning tree

Course Outcome 3(CO3):

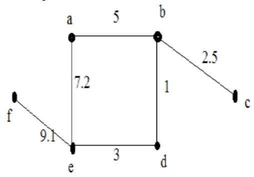
1. Write down Hungarian algorithm
2. How many non-isomorphic 1-factorizations are there of the cube.
3. Write down any three properties of blocks of a connected graph

Course Outcome 4 (CO4):

1. Prove that every k-regular bipartite graph has a perfect matching if k >0.
2. Find the formula to find the number of perfect matchings of K_{2n} and $K_{3,3}$
3. Write down an algorithm to find the maximum matching of a graph

Course Outcome 5 (CO5):

1. Find the clique number for the following graph

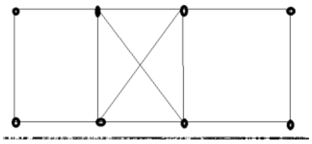


2. Write down an algorithm to find a maximum Clique
3. What is the procedure to find the chromatic polynomial for the given graph?

Course Outcome 6(CO6):

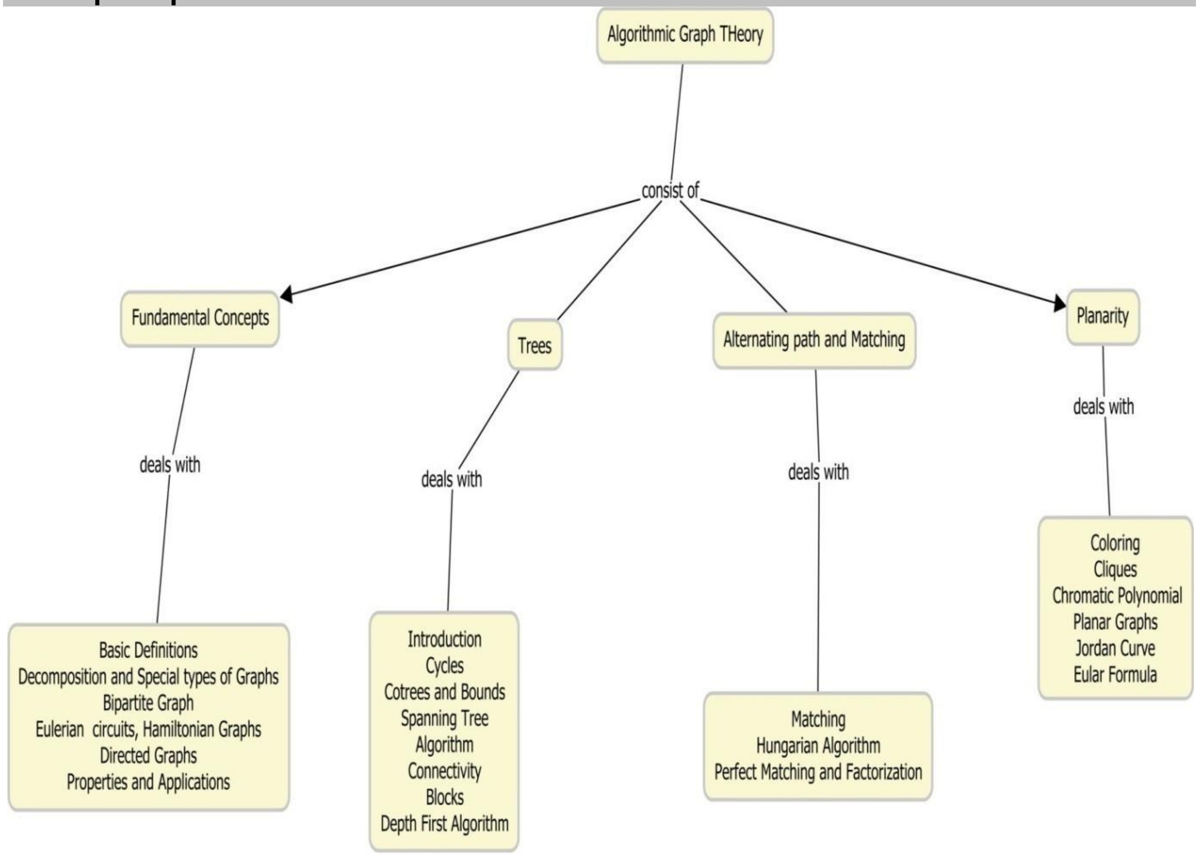
1. Define a planar graph and State Euler's polyhedral formula

2. Check whether the following graph is planar.



3. State Kuratowski's theorems.

Concept Map



Syllabus

Fundamentals Concepts: Graph, Models, Basic definitions – Path, Cycle and Path, Matrices and Isomorphism, Decomposition and special types of graphs. Connections in graphs-Bipartite graph, Eulerian Circuit, Hamiltonian graph .**Directed graphs:** Directed graphs, its Properties and its applications.
Trees, Cycles and Connectivity: Introduction, Fundamental Cycles, Co trees and Bounds, , Spanning Tree , Spanning Tree Algorithm**. Connectivity**: Introduction, Finding the blocks of the graph, Depth First Algorithm.
Alternating Path and Matching: Matching , Hungarian Algorithm, Perfect matching and one factorization. **Coloring:** Coloring, Cliques, Chromatic Polynomial. **Planarity:** Planar Graphs, Jordan Curve, Graph minor and subdivision and Euler's formula.

Learning Resources

1. Introduction to Graph Theory, Douglas B.West , Second Edition Pearson publications 2015.
   * 1.1, 1.2 , 1.4 **(Module -I)**

2. Discrete mathematics and its applications – Graph, Algorithms and Optimization, William Kocay, Donald L. Kreher ,CRC Press 2013**.**
   *** (Module -I)** 1.2, 3.4, 9.1, 9.2
   *** (Module -II)** 4.1- 4.4(upto 4.4.2), 5.7, 6.1 - 6.4
   *** (Module -III)** 7.1-7.3
   ***(Module -IV)** 11.1, 11.2, 11.5, 12.1-12.4

3.Optimization Algorithms for Networks and Graphs,Second Edition - James Evans ,Edward Minieka , Marcel Dekker (1992),CRC PRESS.

4. A Textbook of Graph Theory, Authors: Balakrishnan**,** R., Ranganathan**,** K. second edition.2012, springer,

5. Graph Theory with Applications to Engineering and Computer Science**,** Narsingh Deo**,**PHI Learning

Course Contents and Lecture Schedule

Module No.	Topic	No. of Hours	Course Outcome
1.	**Fundamentals Concepts**		
1.1	Graph, Models, Basic definitions	2	CO1
1.2	Decomposition and special types of graphs	2	C01
1.3.	Connections in graphs-Bipartite graph	2	C01
1.4	Eulerian Circuit, Hamiltonian graph	2	C01
	Directed graphs		

1.5	Directed graphs	2	CO1
1.6	Properties and its applications	2	CO1
2	**Trees, Cycles and Connectivity**		
2.1	Trees & Fundamental Cycles	2	C02
2.2	Co trees and Bounds & Spanning Tree	2	CO2
2.3	Spanning Tree Algorithm	2	CO2
	Connectivity *		
2.4*	Connectivity,Finding the blocks of the graph	2	C03
2.5	Depth First Algorithm	2	CO3
3	**Alternating Path and Matching**		
3.1*	Matching	1	CO4
3.2*	Hungarian Algorithm	2	CO4
3.3	Perfect matching and one factorization	2	CO4
4	**Coloring**		
4.1	Coloring	1	CO5
4.2	Cliques	2	CO5
4.3	Chromatic Polynomial	2	CO5
	Planarity		
4.4	Planar Graphs& Jordan Curve	1	CO6
4.5*	Graph minor and subdivision	2	CO6
4.6	Euler's formula	1	CO6
	Total	36	

*Theorems need not be proved.

Course Designers:

1. Dr.G.Jothilakshmi gjlmat@tce.edu
2. Dr.A.P.Pushpalatha appmat@tce.edu
3. Dr.C.S. Senthilkumar kumarstays@tce.edu

18MAFB0	FUZZY SETS AND CLUSTERING

Category	L	T	P	Credit
FE	3	0	0	3

Preamble

Fuzzy set theory provides a major newer paradigm in modeling and reasoning with uncertainty. Evolution of fuzzy mathematics has added promising new dimensions to the development of research and technology. The main objective of this course is to introduce the basic concepts of fuzzy sets which include representations & operations of fuzzy set, fuzzy logic fuzzy relations and fuzzy classifications.These topics have wide range of applications in engineering, biology, medicine, psychology, economics, and many other disciplines.

Prerequisite

Basic set theory

Course Outcomes

On the successful completion of the course, students will be able to

Course Outcomes		**Weightage in %**
CO1:	Calculate support, heights, normal alpha cuts and strong alpha cuts from the constructed membership functions.	20%
CO2:	Apply extension principle to find image and inverse image of a fuzzy set.	10%
CO3:	Apply basic fuzzy inference and approximate reasoning.	15%
CO4:	Compute fuzzy relations for equivalence and compatibility.	15%
CO5:	Compute projections and cylindrical extensions	15%
CO6:	Experiment clustering and its analysis using Hard c-means and Fuzzy c-means.	25%

CO Mapping with CDIO Curriculum Framework

CO	TCE Proficiency Scale	Learning Domain Level			CDIO Curricular Components
		Cognitive	Affective	Psychomotor	
CO1	TPS2	Understand	Respond	-	1.1.1, 2.1.1, 2.3.1, 2.4.4
CO2	TPS3	Apply	Value	-	1.1.1, 2.1.1, 2.1.3, 2.4.4

CO3	TPS3	Apply	Value	-	1.1.1, 2.1.1, 2.1.2, 2.2.1, 2.4.4
CO4	TPS2	Understand	Respond	-	1.1.1, 2.1.1, 2.4.3, 2.4.4

Mapping with Programme Outcomes and Programme Specific Outcomes

Cos	PO1	PO2	PO3	PO4	PO5	PO6	PO7	PO8	PO9	PO10	PO11	PO12	PSO1	PSO2	PSO3
CO1	S	S	S	-	-	-	-	-	-	M	-	M	M		L
CO2	S	S	S	-	-	-	-	-	-	M	-	M	M		L
CO3	S	S	S	-	-	-	-	-	-	M	-	M	M		L
CO4	S	S	S	-	-	-	-	-	-	M	-	M	M		L

S- Strong; M-Medium; L-Low

Assessment Pattern: Cognitive Domain

Cognitive Levels	Continuous Assessment Tests			Assignment			Terminal Examination
	1	2	3	1	2	3	
Remember	10	10	10	-	-	-	----
Understand	20	20	20	50	50	50	30
Apply	70	70	70	50	50	50	70
Analyse	00	00	00	-	-	-	00
Evaluate	00	00	00	-	-	-	00
Create	00	00	00	-	-	-	00

Course Level Assessment Questions

Course Outcome (CO1):

1. Produce the difference between randomness and fuzziness.
2. Sequence the fuzzy sets defined by the following membership function(x>0) by the inclusion relation:

$A(x) = \dfrac{1}{1+10x}$; $A(x) = \left(\dfrac{1}{1+10x}\right)^{\frac{1}{2}}$; $A(x) = \left(\dfrac{1}{1+10x}\right)^{2}$

3. Consider the fuzzy sets A,B,C defined on the interval X=[0,10] of real numbers by the membership grade functions

   $A(x) = \dfrac{x}{x+2}$; $B(x) = 2^{-x}$; $C(x) = \dfrac{1}{1+10(x-2)^2}$

   Calculate the α -cuts and strong α -cuts , support , height and normal of the three fuzzy sets defined above for α = 0.2,0.5,0.8,1.
4. Explain why the law of contradiction and the law of exclusive middle are violated in fuzzy set theory under the standard fuzzy operations with its significance
5. Show that the following operations satisfy law of excluded middle and the law of contradiction u(a,b) = min(1,a+b), i(a,b) = max(0,a+b-1), c(a)=1-a .
6. Consider the fuzzy sets A,B,C defined on the interval X=[0,10] of real numbers by the membership grade functions

   $A(x) = \dfrac{x}{x+2}$; $B(x) = 2^{-x}$; $C(x) = \dfrac{1}{1+10(x-2)^2}$

7. Produce graphs of the membership grade functions of each of the following:

   (i) $\bar{A}, \bar{B}, \bar{C}$ (ii) AUB,AUC,BUC (iii) $\bar{A}\cap C, \bar{B}\cap A, \bar{C} \cap A$.

Course Outcome (CO2):

1. Let the membership function be $B(x) = 2^{-x}$ be defined on the universal set

   {0,1,2,3,4,5,6,7,8,9,10} and let f(x)=x^2 for all x€X. Compute F(B) using extension principle.
2. Let X={0, 1, 2,.......... 10} and Y={ x^2 / x is an element of X } and f be a crisp function such that $f : X \to Y$ defined by $f(x) = x^2, \ \forall x \in X$. Let D be a fuzzy set on Y such that

   $D = \dfrac{0.5}{4} + \dfrac{0.6}{16} + \dfrac{0.7}{25} + \dfrac{1}{100}$. Identify f^{-1}(D) by means of extension principle.

Course Outcome (CO3):

1. For the inference rule $[(a \to b) \wedge (b \to c)] \to (a \to c)$, demonstrate that the rule is a tautology.
2. Consider the following two discrete fuzzy sets which are defined on the universe X={-5, 5}. $A = Zero = \left\{\dfrac{0}{-2} + \dfrac{0.5}{-1} + \dfrac{1.0}{0} + \dfrac{0.5}{1} + \dfrac{0}{2}\right\}$ &

   $B = Positive\ Medium = A = Zero = \left\{\dfrac{0}{0} + \dfrac{0.6}{1} + \dfrac{1.0}{2} + \dfrac{0.6}{3} + \dfrac{0}{4}\right\}$

   Construct the relation for the rule 'If a, then b'(i.e. if x is zero, then y is positive medium) using Mamdani equation and the product implication equation

$\mu_R(x, y) = \min\{\mu_A(x), \mu_B(y)\}$ and $\mu_R(x, y) = \mu_A(x).\mu_B(y)$

3. The mixing composition of a chemical plant is governed according to a differential equation. But, to approximate this process, we know the following linguistic information. IF the concentration within the tank is high, THEN the tank should drain at a fast rate. The fuzzy sets for a high concentration and fast

drainage rate can be H=high= $\left\{\frac{0}{100}+\frac{0.2}{150}+\frac{0.4}{200}+\frac{0.7}{250}+\frac{1.0}{300}\right\}$ represents

universe X in gms/litre and F=fast= $\left\{\frac{0}{0}+\frac{0.3}{2}+\frac{0.6}{4}+\frac{1.0}{6}+\frac{0.8}{8}\right\}$ represents

universe Y in litres/minute. From these two fuzzy sets construct a relation for the rule using classical implication. Suppose a new rule uses a different concentration say, "moderately high", and is expressed by the fuzzy membership function for this as H' = moderately high= $\left\{\frac{0}{100}+\frac{0.3}{150}+\frac{0.3}{200}+\frac{1.0}{250}+\frac{0.1}{300}\right\}$.

Using max.product composition, identify the resulting drainage rate.

4. In the manufacture of concrete there are two key variables; the water content, measured in percentage of total weight, and the temperature at curing in the batch plant, measured in degrees, Fahrenheit. Nominal water content percentages vary from 1 to 5% and nominal temperature limits are from 40 to 80^0F. We characterize each parameter in fuzzy linguistic terms as below.

Low temperature= $\left\{\frac{1.0}{40}+\frac{0.7}{50}+\frac{0.5}{60}+\frac{0.3}{70}+\frac{0}{80}\right\}$

High temperature= $\left\{\frac{0}{40}+\frac{0.2}{50}+\frac{0.4}{60}+\frac{0.7}{70}+\frac{1.0}{80}\right\}$

High water content= $\left\{\frac{0}{1}+\frac{0.2}{2}+\frac{0.4}{3}+\frac{0.9}{4}+\frac{1.0}{5}\right\}$

Low water content= $\left\{\frac{1.0}{1}+\frac{0.8}{2}+\frac{0.6}{3}+\frac{0.4}{4}+\frac{0.2}{5}\right\}$

Identify the following membership functions

i) Temperature not very low
ii) Temperature not very high
iii) Temperature not very low and very high
iv) Water content slightly high
v) Water content fairly high
vi) Water content not very low or fairly low

Course Outcome (CO4):

1. The fuzzy binary relation R is defined on sets X={1,2,3..,100}. And Y ={ 50,51,…,100} and represents the relation "x is smaller than y. It is defined by the membership function

$$R(x,y)=\begin{cases}1-\dfrac{x}{y}; x\le y\\ 0; otherwise\end{cases}$$ where x € X and y € Y..

(i) What is the domain of R. (ii) What is the range and height of R. (iii) Calculate R^{-1}.

2 Let the relations be defined by the matrices

$$M_3 = \begin{pmatrix} 1 & 0 & 0,7 \\ 0 & 1 & 0 \\ 0.7 & 0 & 1 \end{pmatrix}, M_{12} = \begin{pmatrix} 1 & 0 & 0.8 & 0 & 0.6 & 0.8 & 0 \\ 0 & 1 & 0 & 0.6 & 0 & 0.5 & 0 \\ 0.8 & 0 & 1 & 0.8 & 0 & 0 & 0 \\ 0 & 0.6 & 0.8 & 1 & 0 & 0 & 0.8 \\ 0.6 & 0 & 0 & 0 & 1 & 0.6 & 0 \\ 0.8 & 0.5 & 0 & 0 & 0.6 & 1 & 0 \\ 0 & 0 & 0 & 0.8 & 0 & 0 & 1 \end{pmatrix}.$$

Draw simple diagrams of the relations and compute all complete alpha covers of the relations.

3. Consider the fuzzy matrix $R_1 = \begin{pmatrix} 1 & 0.8 & 0 & 0.1 & 0.2 \\ 0.8 & 1 & 0.4 & 0 & 0.9 \\ 0 & 0.4 & 1 & 0 & 0 \\ 0.1 & 0 & 0 & 1 & 0.5 \\ 0.2 & 0.9 & 0 & 0.5 & 1 \end{pmatrix}$.Experiment how many compositions are required to make the given tolerance relation into fuzzy equivalence relation.

Course Outcome (CO5):

1. Consider the sets X_1={0, 1}, X_2={0, 1} & X_3={0, 1, 2} and the ternary fuzzy relation on $X_1 \times X_2 \times X_3$ given below.

$(0,0,0) \to 0.4,\ (0,0,1) \to 0.9,\ (0,0,2) \to 0.2,\ (0,1,0) \to 1.0,$
$(0,1,1) \to 0, (0,1,2) \to 0.8,\ (1,0,0) \to 0.5,\ (1,0,1) \to 0.3,$ Identify the projections R_1, R_2, R_3.
$(1,0,2) \to 0.1,\ (1,1,0) \to 0,\ (1,1,1) \to 0.5,\ (1,1,2) \to 1$

Also identify the cylindric closure of the above three projections.

2. Consider the matrices

$$M_1 = \begin{bmatrix} 1 & 0 & 0.7 \\ 0.3 & 0.2 & 0 \\ 0 & 0.5 & 1 \end{bmatrix}, M_2 = \begin{bmatrix} 0.6 & 0.6 & 0 \\ 0 & 0.6 & 0.1 \\ 0 & 0.1 & 0 \end{bmatrix} \ \& \ M_3 = \begin{bmatrix} 1 & 0 & 0.7 \\ 0 & 1 & 0 \\ 0.7 & 0 & 1 \end{bmatrix}$$ as pages in

a three dimensional array that represents a fuzzy ternary relation. Determine

a) All two dimentional projections

b) Cylindric extensions and cylindric closure of the two dimentional projections

c) All one dimensional projections

d) Cylindric extensions and cylindric closure of the one-dimentional projections

Course Outcome (CO6):

1. Produce algorithmic steps for K-Means clustering.
2. Produce algorithmic steps for Fuzzy c-Means clustering
3. Produce comparative analysis of Hard c-Means and Fuzzy c-Means Algorithms
4. The biochemical department of a prominent university is conduction research in bone structure. One study involves developing a relationship between the wrist joint angle and the sarcomere length . One study involves developing a relationship between the wrist joint angle and the sarcomere length in the lowest arm. In this study the following data were obtained.

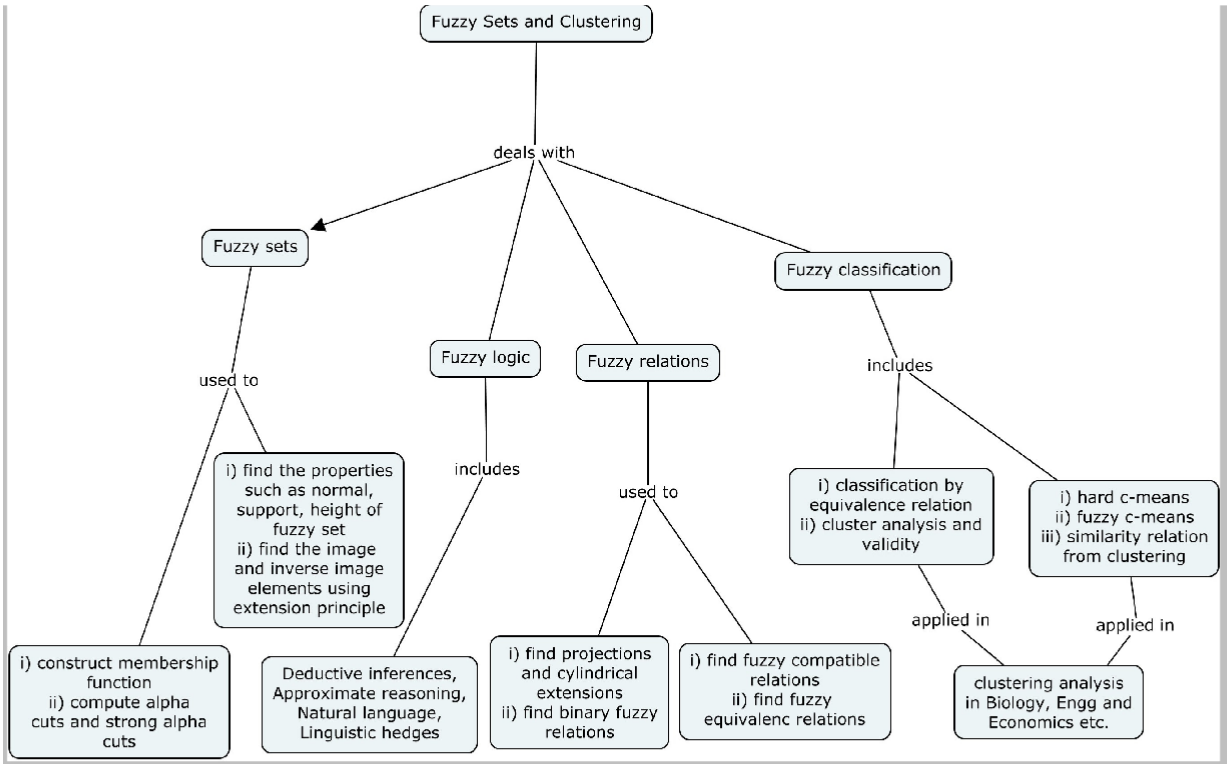
Wrist Joint Angle (deg)	-75	-50	-25	0	25
Sarcomere Length (µm)	3	3.25	3.5	2.75	3

(a) Classify these data, into one cycle , into two classes using the hard c-means method.

(b) Classify these data into two classes using the fuzzy c-means method; use m'=2 and €=0.01 and conduct two cycles.

(c) Calculate the classification metric.

(d) Compute the similarity relation for the U-partition from part (b).



Syllabus

Fuzzy Sets: Basic Types – Basic Concepts – α-Cuts – Additional Properties of α-Cuts – Representations of Fuzzy Sets - Extension principle for Fuzzy Sets, Basic Concepts of Fuzzy Numbers. **Fuzzy Logic:** Classical logic-Deductive inferences-Fuzzy logic-Approximate reasoning-Other forms of implication operation-Natural language-Linguistic hedges-Fuzzy rule based systems-Aggregation of fuzzy rules.
Fuzzy Relations: Crisp Versus Fuzzy Relations – Projections and Cylindrical Extensions – Binary Fuzzy Relations – Fuzzy Equivalence Relations – Fuzzy Compatibility Relations.
Fuzzy Classification : Classification by Fuzzy Equivalence relations - Cluster Analysis - Cluster Validity – C-means Clustering - Hard C-means - Fuzzy C-means - Hardening the Fuzzy C-Partitions – Similarity Relations from Clustering.

Learning Resources

1. George J.Klir and Bo Yuan, Fuzzy Sets and Fuzzy Logic, Prentice Hall of India, New Delhi, 2004.
2. Timothy J.Ross, Fuzzy Logic with Engineering Applications, second edition, John Wiley & Sons Pvt. Ltd, 2005.
3. H.J. Zimmermann, Fuzzy Set Theory and its Applications, Allied Publishers Limited, New Delhi, 1991.
4. https://onlinecourses.nptel.ac.in/noc19_ma31/course

Course Contents and Lecture Schedule

Module No.	Topic	No. of Lectures
1	**Fuzzy Sets**	
1.1	Basic Types & Basic Concepts	2
1.2	α-cuts	1
1.3	Additional Properties of α- Cuts	1
1.4	Representations of Fuzzy Sets	1
1.5	Extension Principle for Fuzzy Sets	2
1.6	Basic Concepts of Fuzzy Numbers	2
2	**Fuzzy Logic**	
2.1	Classical logic, Deductive inferences	1
2.2	Fuzzy logic, Approximate reasoning	1
2.3	Other forms of implication operation, Natural language	2
2.4	Linguistic hedges, Fuzzy rule based systems	2
2.5	Aggregation of fuzzy rules	2
3	**Fuzzy Relations**	
3.1	Crisp Versus Fuzzy Relations	2
3.2	Projections and Cylindrical Extensions	1
3.3	Binary Fuzzy Relations	2
3.4	Fuzzy Compatibility Relations	2
3.5	Fuzzy Equivalence Relations	2
4	**Fuzzy Classification**	
4.1	Classification by Fuzzy Equivalence Relations	2
4.2	Cluster Analysis - Cluster Validity	1
4.3	C-means Clustering	2
4.4	Hard C-Means , Fuzzy C-Means-	3
4.5	Hardening the Fuzzy C-Partitions	1
4.6	Similarity Relations from Clustering.	1
	Total hours	**36**

Course Designer:

Dr. M. Kameswari mkmat@tce.edu

Dr.T. Lakshmi tlakshmimdu@tce.edu

18MAFE0	NUMERICAL LINEAR ALGEBRA

Category	L	T	P	Credit
BS	3	0	0	3

Preamble

A general theory of Mathematical systems involving addition and scalar multiplication of vectors has applications in all Engineering field. Mathematical systems of this form are called Vector spaces or linear spaces. Linear systems of equations are associated with many problems in Engineering and Sciences, as well as with applications of mathematics to the social sciences and quantitative study of business and economic problems. The modules I and II of this subject deal with the concepts on Vector spaces and orthogonality.

Numerical methods are becoming more and more important in engineering applications, because of the difficulties encountered in finding exact analytical solutions. Modules III & IV deal with the Numerical methods for finding the solution of system of linear algebraicequations and also finding all the eigen values and eigen vectors of the given square matrix.

Prerequisite

18MA210

Course Outcomes

On the successful completion of the course students will be able to

CO Number	Course Outcome Statement	Weightage*** in %
CO1	Verify whether the given set is a vector space or not. If so, determine its dimension. Test the consistency of system of linear equation using rank of a matrix	15%
CO2	Predict an orthonormal basis and fit a curve by the method of least square method.	15%
CO3	Solve the system of linear equations numerically	10%
CO4	Compute eigen values and eigen vectors using Power method and Jacobi method	20%
CO5	Decompose a given matrix using QR, LU and Cholesky methods	20%
CO6	Compute Pseudo inverse for the given matrix using SVD method	20%

CO Mapping with CDIO Curriculum Framework

CO #	TCE Proficiency Scale	Learning Domain Level			CDIO Curricular Components (X.Y.Z)
		Cognitive	Affective	Psychomotor	
CO1	TPS3	Apply	Value	-	1.1.1, 1.2.7, 2.1.1
CO2	TPS3	Apply	Value	-	1.1.1, 1.2.7, 2.1.1
CO3	TPS3	Apply	Value	-	1.1.1 , 1.2.7, 2.1.1
CO4	TPS3	Apply	Value	-	1.1.1, 1.2.7, 2.1.1
CO5	TPS3	Apply	Value	-	1.1.1, 1.2.7, 2.1.1
CO6	TPS3	Apply	Value	-	1.1.1, 1.2.7, 2.1.1

Mapping with Programme Outcomes and Programme Specific Outcomes

Cos	PO1	PO2	PO3	PO4	PO5	PO6	PO7	PO8	PO9	PO10	PO11	PO12	PSO1	PSO2	PSO3
CO1	S	M	M	M	-	-	-	-	M	-	-	M			
CO2	S	S	S	S	-	-	-	-	M	-	-	M			
CO3	S	S	S	S	-	-	-	-	-	-	-	M			
CO4	S	S	S	S	-	-	-	-	-	-	-	M			

CO5	S	S	S	S	-	-	-	-	-	-	-	M			
CO6	S	S	S	S	-	-	-	-	-	-	-	M			
CO7	S	S	S	S	-	-	-	-	-	-	-	M			

S- Strong; M-Medium; L-Low

Assessment Pattern: Cognitive Domain

Cognitive Levels	Continuous Assessment Tests			Assignment			Terminal Examination
	1	2	3	1	2	3	
Remember	10	10	10	-	-	-	-
Understand	20	20	20	-	-	-	30
Apply	70	70	70	100	100	100	70
Analyse							
Evaluate							
Create							

Sample Questions for Course Outcome Assessment**

** (2 to 3 at the cognitive level of course outcome)

Course Outcome 1(CO1):

1. Let C be the set of complex numbers. Define addition on C by $(a+bi)+(c+di)=(a+c)+(b+d)i$ And define scalar multiplication by $\alpha(a+bi)=\alpha a+\alpha bi$ For all real number α. Show that C is a vector space with these operations.
2. Show that $R^{m\times n}$,together with the usual addition and scalar multiplication of matrices, satisfies the eight axioms of a vector space.
3. Find the values of a and b so that the rank of the matrix $\begin{pmatrix} 1 & 1 & 2 \\ 0 & 1 & 0 \\ 2 & a & b \end{pmatrix}$ is 2

Course Outcome 2(CO2):

1. Show that $\left\{ \frac{(1,1,1)^T}{\sqrt{3}}, \frac{(2,1,-3)^T}{\sqrt{14}}, \frac{(4,-5,1)^T}{\sqrt{42}} \right\}$ is an orthonormal set in R^3
2. Consider the vector space $C[-1,\ 1]$ with inner product defined by $< f,g >= \int_{-1}^{1} f(x)g(x)dx$ Calculate orthonormal basis for subspace spanned by $\{1, x, x^2\}$
3. Construct an orthonormal basis for P_3 if the inner product on P_3 is defined by $\langle p,q\rangle=\sum_{i=1}^{3} p(x_i)q(x_i)$ where $x_1=-1, x_2=0$ & $x_3=1$corresponding to the basis $\{1,x,x^2\}$ using Gram-Schmidt orthogonalization process.

Course Outcome 3(CO3):

1. Solve the system of equations $4x+2y+z=14,\ x+5y-z=10, x+y+8z=20$ by Gauss-Seidel method.

2. Solve by using Gauss -Seidal method and Gauss Jacobi method also.

10x - 2y -z -w =3 ; -2x +10 y -z -w =15 ; -x -y +10z -2w =27 ; -x -y-2z+10w = - 9

3. Solve by Gauss Jacobi Method 17 x +65 y -13 z +50w =84 ; 12x +16y +37 z +18w =25 ; 56 x +23y +11z -19w =36 ; 3x -5y +47 z +10 w = 18.

Course Outcome 4 (CO4):

1. Compute the largest eigen value of $A = \begin{pmatrix} 2 & 0 & 0 \\ 0 & 2 & 1 \\ 0 & 1 & 2 \\ 0 & 0 & 0 \end{pmatrix}$ by using power method and its corresponding eigen vector

2. Compute all the eigen values and eigen vectors of $A = \begin{pmatrix} -1 & 1 & 0 \\ 2 & 0 & 1 \\ 0 & 1 & 1 \\ 3 & -1 & 1 \end{pmatrix}$ by power method

3. Compute all the eigen values and eigen vectors of A = $\begin{bmatrix} -4 & 2 & 2 \\ 3 & -3 & 3 \\ 6 & 6 & 0 \end{bmatrix}$ by Jacobi method

Course Outcome 5 (CO5):

1. Construct a QR decomposition for the matrix $A = \begin{pmatrix} 4 & 2 & 1 \\ 2 & 0 & 1 \\ 2 & 0 & -1 \\ 1 & 2 & 1 \end{pmatrix}$

2. Construct a QR decomposition for the matrix $\begin{bmatrix} -4 & 2 & 2 \\ 3 & -3 & 3 \\ 6 & 6 & 0 \end{bmatrix}$

3. Solve the system of equations $4x - y = 1, -x + 4y - z = 0, -y + 4z = 0$ by Cholesky method. Also find the inverse of the coefficient matrix.

Course Outcome 6(CO6):

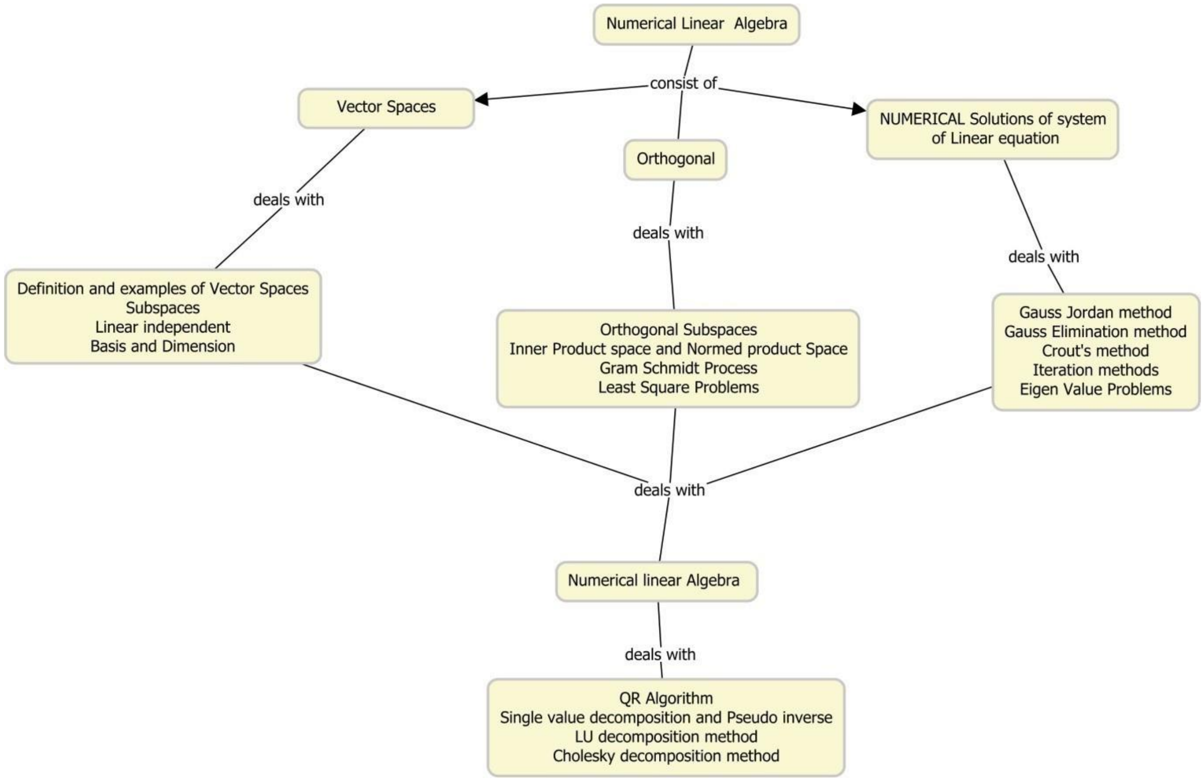
1. Obtain Singular Value Decomposition for the matrix $A = \begin{pmatrix} 2 & 0 & 0 \\ 0 & 2 & 1 \\ 0 & 1 & 2 \\ 0 & 0 & 0 \end{pmatrix}$ and hence find A^{-1}.

2. Obtain singular value decomposition for the matrices A = $\begin{pmatrix} 1 & 1 \\ 1 & 1 \\ 0 & 0 \end{pmatrix}$ and B= $\begin{pmatrix} 1 & 3 \\ 3 & 1 \\ 0 & 0 \\ 0 & 0 \end{pmatrix}$

hence find its pseudo inverse

3. Obtain pseudo inverse for the matrix4. Compute a singular value decomposition for the matrix $\begin{bmatrix} 2 & 2 & -2 \\ 2 & 2 & -2 \\ -2 & -2 & 6 \end{bmatrix}$

Concept Map



Syllabus

Vector Spaces: Definition and examples, Subspaces, linear independence, basis and dimension. Consistency of system of equations **Orthogonality:** Orthogonal subspaces, Inner Product spaces, and Gram Schmidt orthogonalization process, Least square Problems. **Numerical Methods:** Numerical solutions of system of equations: Gauss Jordan method, Gauss Elimination method, Crout's method. **Iteration method**: Gauss Jacobi, and Gauss Seidal method. **Eigen value problem**: Power method, Jacobi method. **Decomposition Methods:** QR decomposition, Singular value decomposition , LU decomposition , Pseudo inverse and Cholesky decomposition

Learning Resources

1. Steven J. Leon, "Linear Algebra with Applications", Macmillan publishing company, New York, 1990.
2. S. R. K. Iyengar, R. K. Jain, Mahinder Kumar Jain, "Numerical methods for Scientific and Engineering Computations", New Age International publishers, 6th Edition, 2012.
3. https://nptel.ac.in/content/syllabus_pdf/111106051.pdf

Course Contents and Lecture Schedule

Module No.	Topic	No. of Hours	Course Outcome
1.	**Vector Spaces**		
1.1	Vector spaces	2	CO1
1.2	Sub-spaces	1	CO1
1.3.	Linear independence and dependence	2	CO1
1.4	Basis and dimension; examples	2	CO1
1.5	Consistency of system of equations	2	CO1
2	**Orthogonality**		
2.1	Orthogonal subspaces	2	CO2
2.2	Inner product space,	3	CO2
2.3	Gram Schmidt orthonormalisation process	2	CO2
2.4	Least square Problems	2	CO2
3	**Numerical Method**		
3.1	**Numerical solutions of system of algebraic linear equations**: Gauss Jordan method	1	CO3
3.2	Gauss Elimination method	1	CO3
3.3	Crout's method	2	CO3
3.4	**Iteration method**: Gauss Jacobi, and Gauss Seidal method.	2	CO3
3.5	**Eigen value problem**: Power method, Jacobi method	3	CO4
4	**Decomposition Methods:**		
4.1	QR algorithm	2	CO5
4.2	LU decomposition method	2	CO5
4.3	Cholesky decomposition method	2	CO5
4.4	Singular value decomposition and Pseudo inverse	3	CO6
	Total	36	

Course Designers:

1. Dr.G.Jothilakshmi gjlmat@tce.edu
2. Dr.A.P.Pushpalatha appmat@tce.edu
3. Dr.M.Mutharasan mmmat@tce.edu

18MAFG0	**MATHEMATICAL FOUNDATIONS FOR MACHINE LEARNING**

Category	L	T	P	Credit
PC	3	0	0	3

Preamble

Machine learning focuses on the development of computer program that can access data and use it learn for themselves. Heterogeneous data are used while learning. Storage of that data is a challenging one. Dimensionality reduction is one of the major techniques required for data storage. It is the process of reducing the dimension of the feature set. This course aims to present the basic mathematical knowledge for dimensionality reduction. Dimensionality reduction requires principal component analysis, independent component analysis which in turn requires vectors, orthogonal projection, Gram Schmidt, orgonalization process, singular value decomposition. These topics are provided as different modules in this course.

Prerequisite

Nil

Course Outcomes

On the successful completion of the course students will be able to

CO Number	Course Outcome Statement	Weightage*** in %
CO1	Differentiate between homogeneous and heterogeneous data received from the data source	5%
CO2	Construct Length, Distance, Modulus, inner product 15%	15%
CO3	Apply norms for constructing Average, RMS, S.D,Correlation	15%
CO4	Build basis, linear dependence, independence for the corresponding problem	15%
CO5	Develop Gram-Schmidt Orthogonalization Process	20%
CO6	Build projections from higher dimensional subspaces to lower dimensional subspaces	10%
CO7	Model Data reduction using SVD,PCA,ICA	20%

CO Mapping with CDIO Curriculum Framework

CO #	TCE Proficiency Scale	Learning Domain Level			CDIO Curricular Components (X.Y.Z)
		Cognitive	Affective	Psychomotor	
CO1	TPS2	Understand	Respond	-	1.2,2.3.1,2.3.2
CO2	TPS3	Apply	Value	-	1.2,2.3.1,2.3.2,2.3.3,2.3.1.4, 3.1.5
CO3	TPS3	Apply	Value	-	1.2,2.3.1,2.3.2,2.3.3,2.3.2.3,
CO4	TPS3	Apply	Value	-	1.2,2.3.1,2.3.2,2.3.3,2.3.4
CO5	TPS3	Apply	Value	-	1.2,4.3.
CO6	TPS3	Apply	Value	-	1.2,4.3.4
CO7	TPS3	Apply	Value	-	.2,2.3.1,2.3.2,2.3.3,2.3.4,

Mapping with Programme Outcomesand Programme Specific Outcomes

COs	PO1	PO2	PO3	PO4	PO5	PO6	PO7	PO8	PO9	PO10	PO11	PO12
CO1	M											S
CO2	S	M		L						L		S
CO3	S	M	L	M	M			L				S
CO4	S	M	M	L	M			L				S
CO5	M	L										S
CO6	M	L										S
CO7	S	M	M	M								S

S- Strong; M-Medium; L-Low

AssessmentPattern: Cognitive Domain

Cognitive Levels	Continuous Assessment Tests			Assignment			Terminal Examination
	1	2	3	1	2	3	
Remember	30	30	30	-	-	-	20
Understand	30	30	30	-	-	-	30
Apply	40	40	40	100	100	100	50
Analyse	0	0	0				0
Evaluate	0	0	0				0
Create	0	0	0				0

Assessment Pattern: Psychomotor

Psychomotor Skill	Miniproject/Assignment/Practical Component
Perception	-
Set	-
Guided Response	-
Mechanism	-
Complex Overt Responses	-
Adaptation	-
Origination	-

Sample Questions for Course Outcome Assessment**

Course Outcome 1 (CO1):

1. Differentiate between homogeneous and heterogeneous data
2. Identify the need for mathematics in Machine learning
3. State the data source for homogeneous and heterogeneous data

Course Outcome 2 (CO2):

1. Find the length of the vectors $v = \langle 1,3,4 \rangle$

2. Find the angle between the vectors $u = 3i + 2j + k$ and $v = 2i + 3j$.

3. Use the Euclidean inner product in R^3 to find the orthogonal projection of **u** = (6, 2, 4) onto **v** = (1, 2, 0).

Course Outcome 3 (CO3):

1. RMS value and average of block vectors. Let x be a block vector with two vector elements, $x = (a,b)$, where a and b are vectors of size n and m, respectively.

   a) Express rms(x) in terms if rms(a), rms(b) m, ans n.
   b) Express avg(x) in terms if avg(a), m, and n.
2. Neighboring electronic health records. Let $x_1, ..., x_N$ be n-vectors that contain n

   features extracted from a set of N electronic health records (EHRs), for a population of N patients. (The features might involve patient attributes and current and past symptoms, diagnoses, test results, hospitalizations, procedures, and medications.)

3. Briefly describe in words a practical use for identifying the 10 nearest neighbors of a given EHR (as measured by their associated feature vectors), among the other EHRs

	Veterans Day	Memorial Day	Academy Awards	Golden Globe Awards	Super Bowl
Veterans Day	0	60.6	85.7	87.0	87.7
Memorial Day	60.6	0	85.6	87.5	87.5
Academy Awards	85.7	85.6	0	58.7	85.7
Golden Globe Awards	87.0	87.5	58.7	0	86.0
Super Bowl	87.7	87.5	86.1	86.0	0

Calculate Average, Standard Deviation, Correlation.

Course Outcome 4(CO4):

1. Show that $1, x, x^2, x^3$ are linearly independent in $C(-\infty, \infty)$
2. If $v_1, v_2, v_3, \ldots. v_n$ are elements of vector space V, then Span $(v_1, v_2, v_3, \ldots. v_n)$ is a subspace of V.
3. Prove that the subset of a linearly independent set is linearly independent

Course Outcome 5 (CO5):

1. Find an orthonormal basis for P_3 if the inner product on P_3 is defined by

$$\langle p, q \rangle = \sum_{i=1}^{3} p(x_i)q(x_i)$$

Where $x_1 = -1, x_2 = 0, \quad and \ x_3 = 1.$

2. Let $A = \begin{pmatrix} 1 & -1 & 4 \\ 1 & 4 & -2 \\ 1 & 4 & 2 \\ 1 & -1 & 0 \end{pmatrix}$ Find an orthonormal basis for the column space of A.

3. Given the basis $\{(1,2,-2)^T, (4,3,2)^T, (1,2,1)^T\}$ for R^3 , use the Gram-Schmidt process to obtain an orthonormal basis.

Course Outcome 6 (CO6):

1. Find the orthogonal projection of $y = (2,3)$ onto the line $L = \langle (3,1) \rangle$

2. Let Π be the plane spanned by vectors v1 = (1, 1, 0) and v2 = (0, 1, 1). (i) Find the orthogonal projection of the vector x = (4, 0, −1) onto the plane Π. (ii) Find the distance from x to Π.

3. Let L be the line in R^3 that consists of all scalar multiples of the vector $\begin{bmatrix} 2 \\ -2 \\ -1 \end{bmatrix}$ Find the orthogonal projection of the vector $v = \begin{bmatrix} 5 \\ 3 \\ \end{bmatrix}$

Course Outcome 7 (CO7):

1. **1.** Find the singlular value decomposition of each of the following matrices:

a) $\begin{pmatrix} 1 & 1 \\ 2 & 2 \end{pmatrix}$ b) $\begin{pmatrix} 2 & -2 \\ 1 & 2 \end{pmatrix}$

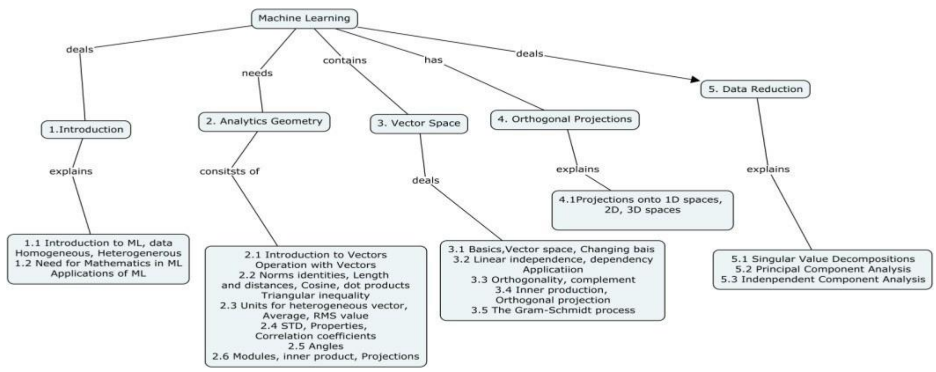
2. Find the singlular value decomposition of each of the following matrices:

c) $\begin{pmatrix} 1 & 3 \\ 3 & 1 \\ 0 & 0 \\ 0 & 0 \end{pmatrix}$ d) $\begin{pmatrix} 2 & 0 & 0 \\ 0 & 2 & 1 \\ 0 & 1 & 2 \\ 0 & 0 & 0 \end{pmatrix}$

3. Perform PCA & ICA for the data

	0	1	2	3	4	5	6	7	8	9	10	11	12	13
0	1	14.23	1.71	2.43	15.6	127	2.80	3.06	0.28	2.29	5.64	1.04	3.92	1065
1	1	13.20	1.78	2.14	11.2	100	2.65	2.76	0.26	1.28	4.38	1.05	3.40	1050
2	1	13.16	2.36	2.67	18.6	101	2.80	3.24	0.30	2.81	5.68	1.03	3.17	1185
3	1	14.37	1.95	2.50	16.8	113	3.85	3.49	0.24	2.18	7.80	0.86	3.45	1480
4	1	13.24	2.59	2.87	21.0	118	2.80	2.69	0.39	1.82	4.32	1.04	2.93	735

Concept Map



Syllabus

Machine Learning: Introduction to Machine learning, data – homogeneous, heterogeneous, Data Source, need for mathematics in machine learning - application of machine learning.

Analytic Geometry : Introduction to vectors , Operations with vectors - Norms, Norm Identities, Norm of a sum, Lengths and Distances, cosine & dot product, Triangular Inequality, Units for Heterogeneous Vector entries, Average, RMS Value, Standard Deviation, Properties of Standard Deviation, Angle between two vectors, Acute and Obtuse angles, Correlation Coefficient, Modulus & inner product, projection, rotations.

Vector Spaces: Vector space, basis, dimensions charging basis, linear independence & Linear dependence of a set of vectors- Applications in changing basis, Orthogonality ,orthonormal basis , orthogonal complement , inner product of functions , orthogonal projections , The Gram –Schmidt process.

Orthogonal Projections: Projection onto 1D subspaces , 3D data onto a 2D subspace, higher dimensional subspaces.

Data Reduction: Singular Value Decomposition, Principal Component Analysis, Independent Component Analysis.

Learning Resources

1. Stephen Boyd Department of Electrical Engineering Stanford University, Lieven Vandenberghe , Department of Electrical and Computer Engineering, University of California, Los Angeles, " Introduction to Applied Linear Algebra", © Cambridge University Press, 2018
2. Jean Gallier and Jocelyn Quaintance, Department of Computer and Information Science University of Pennsylvania Philadelphia," Linear Algebra for Computer Vision, Robotics, and Machine Learning", Jean Gallier August 7, 2019
3. Marc Peter Deisenroth , A. Aldo Faisal, Cheng Soon Ong," Mathematics for Machine Learning ",To be published by Cambridge University,2019.
4. Jason Brownlee, "Basics of Linear Algebra for Machine Learning", © Copyright 2018
5. Steven J. Leon, "Linear Algebra with Applications", Macmillan publishing company, New York, 1990.
6. https://www.coursera.org/lecture/linear-algebra-machine-learning/summary-L0jsf
7. towardsdatascience.com

Course Contents and Lecture Schedule

Module No.	Topic	No. of Hours	Course Outcome
1	**Machine Learning**:		
1.1	Introduction to Machine learning, data – homogeneous, heterogeneous, Data Source	2	CO1
1.2	Need for mathematics in machine learning - application of machine learning.	1	CO1
2	**Analytic Geometry :**		
2.1	Introduction to vectors , Operations with vectors	2	CO2
2.2	Norms Norm Identities, Norm of a sum, Lengths and Distances, cosine & dot product, Triangular Inequality	2	CO2
2.3	Units for Heterogeneous Vector entries, Average, RMS Value	2	CO3
2.4	Standard Deviation, Properties of Standard Deviation, Correlation Coefficient	2	CO3
2.5	Angle between two vectors, Acute and Obtuse angles,	1	CO2
2.6	Modulus & inner product, projection, rotations	2	CO2
3	**Vector Spaces:**		
3.1	Vector space, basis, dimensions ,changing basis	2	CO4
3.2	Linear independence & Linear dependence of a set of vectors- Applications in changing basis	2	CO4
3.3	Orthogonality , orthonormal basis , orthogonal complement	**2**	CO5
3.4	Inner product of functions , Orthogonal projections	2	CO5
3.5	The Gram –Schmidt process	2	CO5
4	**Orthogonal Projections**:		

4.1	Projection onto 1D subspaces , 3D data onto a 2D subspace	2	CO6
4.2	Higher dimensional subspaces	1	CO6
5.	**Data Reduction**:		
5.1	Singular Value Decomposition	3	CO7
5.2	Principal Component Analysis	3	CO7
5.3	Independent Component Analysis	3	CO7
	Total Hours	36	

Course Designers:

1. M.Kameswari mkmat@tce.edu
2. S.Md.Mansoorroomi smmroomi@tce.edu
3. R.Suganya rsuganya@tce.edu